Эпистемология и философия науки
2020. Т. 57. № 3. С. 75–86
УДК 161.201

Epistemology & Philosophy of Science
2020, vol. 57, no. 3, pp. 75–86
DOI: https://doi.org/10.5840/eps202057343

THE MODAL SQUARE OF OPPOSITION AND THE MENTAL MODELS THEORY*

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The mental models theory is a current approach trying to account for human thought and hence communication by highlighting the action of semantics and ignoring, to a large extent, syntax. However, it has been argued that the theory actually contains an underlying syntax related to any kind of modal logic. This paper delves into this last idea and is intended to show that the concepts of possibility and necessity as understood in it fulfill the basic requirement that, according to Fitting and Mendelsohn, every modal logic has to meet: to satisfy the relationships provided by the Aristotelian modal square of opposition.

***Keywords*:** Aristotelian square of opposition, mental models, modal logic, semantics, syntax

ЛОГИЧЕСКИЙ КВАДРАТ И ТЕОРИЯ МЕНТАЛЬНЫХ МОДЕЛЕЙ

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Теория ментальных моделей – это современный подход к объяснению человеческого мышления и коммуникации с упором на семантику и пренебрежением синтаксисом. Однако уже было показано, что эта теория фактически содержит синтаксис, имеющий отношение к модальной логике любого вида. В статье подробно рассматривается данная идея. Автор намерен показать, что понятия возможности и необходимости – в том виде, как они понимаются в этой теории – выполняют требование, которому, согласно Фиттингу и Мендельсону, должна соответствовать всякая модальная логика: удовлетворять отношениям, установленным логическим квадратом Аристотеля.

***Ключевые слова*:** логический квадрат, ментальные модели, модальная логика, семантика, синтаксис

* This paper is a result of the Project CONICYT/FONDECYT/REGULAR/FOLIO No. 1180013, "Recuperación de las formas lógicas de los enunciados a partir de un análisis de las posibilidades semánticas a las que hacen referencia", supported by FONDECYT (National Fund for Scientific and Technological Development), Government of Chile.

Introduction

A truly important framework about rationality and, therefore, the deep human inferential and linguistic activity is clearly at present that given by the mental models theory [e.g., Khemlani et al., 2018]. Maybe one of its most relevant distinguishing features is the fact that it explicitly rejects the idea that some type of formal logic plays some natural role in the human mind [e.g., Johnson-Laird, 2010], which in turn can lead one to think that this approach is also incompatible with the thesis of theories such as the one of the mental logic [e.g., Braine; O'Brien, 1998], that is, with the thesis that there is a real syntax of thought [for an explanation of why the mental models theory is clearly inconsistent with this last thesis, see also, e.g. López-Astorga, 2017]. However, some papers have tried to demonstrate exactly the opposite, and, in particular, have attempted to link the general assumptions of the mental models theory to a system such as system K in modal logic [e.g., Ibid.].

In this way, the main aim of this paper is to keep looking into the idea of a possible relationship between the mental models theory and modal logic, although without proposing equivalence as strong as the one that implies the link to system K. Thus, the goal is just to show that the mental models theory defines 'possibility' and 'necessity' in a manner that it fulfills the minimal condition that, following Fitting and Mendelsohn [1998], needs to be met by every modal logic: to be coherent with relationships such as those indicated in the Aristotelian modal square of opposition.

To achieve this goal, firstly the way the mental models theory understands the concepts of possibility and necessity –and relates them to linguistic activity and the meaning of the sentences expressed in natural language – will be commented on. Then, based on the laws of the Aristotelian square of opposition as presented by Peter of Spain in *Tractatus* (*Summulae Logicales*), the specific modal square of opposition will be analyzed from what is provided both in Περὶ Ἑρμηνείας (*De Interpretatione*), 12 and 13, by Aristotle and in the book by Fitting and Mendelsohn [1998]. Thirdly, it will be shown that, indeed, the approach about possibility and necessity offered by the mental models theory exactly match the relationships between those two alethic qualifiers indicated in the modal square as reproduced by Fitting and Mendelsohn [1998]. Accordingly, the next section is devoted to the way the mental models theory interprets both possibility and necessity.

Possibility and Necessity in the Mental Models Theory

If one pays attention to all the literature on the mental models theory, one can note that its proponents have dealt with all the usual connectives in classical logic [see, e.g., Johnson-Laird, 2012]. Nevertheless, to develop the main aim here, perhaps it is enough to address some of its essential notions and theses, and then, as examples, the cases of two connectives: the conditional and disjunction.

Starting with some of the fundamental principles of the theory, works such as, for instance, the one of Johnson-Laird [2012] indicate them. It can be said that the theory proposes that people do not reason in accordance with classical logic because they do not follow what the truth tables in that logic provide, but mental models that are iconic. This concept of iconicity refers to the fact that the models tend to reproduce reality in the most exact possible way, which, in practice, means that, as shown below, each sentence is linked to one or several models that iconically represent different possibilities of reality in the human mind.

But maybe the key is that, because many times those models match the situations in which a connective is true in classical logic, one might believe that human reasoning is absolutely logical. However, according to the mental models theory, it is not, since, as it will be accounted for, there are also circumstances in which the models do not match the cases in which the particular connective in the sentence is true in its truth table. All of this is easier to see by reviewing the two examples mentioned: the conditional and disjunction [in addition, for further explanations, see also, e.g., Ibid.].

According to the mental models theory, there is no doubt that conditional sentences whose correct interpretation is the one corresponding to them in classical logic can be found. That is the case of, for example, this sentence:

[I] "If he promised then he must take the kids to the zoo" [Johnson-Laird; Byrne, 2002: 663, Table 4].

Following Johnson-Laird and Byrne [2002], a sentence such as this one refers to three possibilities:

[II]

1. [He promised to take the kids to the zoo] is true and [he takes the kids to the zoo] is true.
2. [He promised to take the kids to the zoo] is false and [he takes the kids to the zoo] is true.
3. [He promised to take the kids to the zoo] is false and [he takes the kids to the zoo] is false.

Obviously, the only scenario that would not be allowed would be a situation in which he promised to take the kids to the zoo and he does not take them there, which reveals that, certainly, [I] can be true in exactly the same cases in which the conditional is so in classical logic (i.e., whenever the antecedent is false or the consequent is true).

And something similar is what happens with disjunction. It is not hard to find disjunctive sentences that are true in the cases indicated by classical logic too. For example,

[III] "…Pat is here or Viv is here, or both" [Orenes; Johnson-Laird, 2012: 362].

Clearly, following Orenes and Johnson-Laird [2012], the possibilities corresponding to [III] are:

[IV]

1. [Pat is here] is true and [Viv is here] is false.
2. [Pat is here] is false and [Viv is here] is true.
3. [Pat is here] is true and [Viv is here] is true.

Which means that, indeed, [III] is true in the cases in which a disjunction is so in standard propositional logic (whenever its two disjuncts are not false at the same time).

Nonetheless, this is not always in this way, that fact being the key to explain why human thought does not often match what is expected in accordance with classical logic. Many times the meanings and pragmatics cause the possibilities to be modified. As far as the conditional is concerned, cases such as, for example, the following one are possible:

[V] "If there is gravity (which there is) then your apples may fall" [Johnson-Laird; Byrne, 2002: 663, Table 4].

Which only really enables two possibilities:

[VI]

1. [There is gravity] is true and [your apples fall] is true.
2. [There is gravity] is true and [your apples fall] is false.

[V] has been analyzed in many papers after Johnson-Laird and Byrne [2002] presented it as an example. However, what is important now is that [VI].1 and [VI].2 are the only possibilities in [VI] because the other two scenarios that can be thought cannot be accepted, since they are scenarios in which gravity is not real.

Furthermore, what occurs with disjunction is not very different. Sentences such as this one are also possible:

[VII] "Lucia wore the bracelet or she wore jewelry" [Orenes; Johnson-Laird, 2012: 363].

Now, the possibilities are not equivalent to those in [IV], but

[VIII]

1. [Lucia wore the bracelet] is true and [Lucia wore jewelry] is true.
2. [Lucia wore the bracelet] is false and [Lucia wore jewelry] is true.

[VII] has been studied in many works as well after being used by Orenes and Johnson-Laird [2012], for example, as it happens, in López-

Astorga [2017]. Nevertheless, the point here is also that [VIII].1 and [VIII].2 are the only possibilities in [VII] because, if Lucia wore the bracelet, she necessarily wore jewelry too.

By means of facts such as these ones, the mental models theory usually accounts for the results in empirical experiments that seem to prove that human reasoning is not logical (at least, if classical logic is the criterion). Just one example is to be found in Orenes and Johnson-Laird (2012). Indeed, in that paper, it is reminded that a very controversial logical rule is the one that allows deriving, as a conclusion, a disjunction including a premise as one of its disjuncts. Thus, that rule would enable, for example, to draw a sentence such as [III] from the simple fact that ‘Pat is here’. However, as experimentally shown by Orenes and Johnson-Laird (2012), that is something that people rarely do. There is no doubt that this needs to be explained, especially because the problem is even more complex. Certainly, as the experiments in that very paper reveal, when the disjunction is such as [VII] and the premise indicates something similar to the fact that Lucia was wearing jewelry, what happens is exactly the opposite. In these cases, people often make the inference.

The problem is hence that there is a rule that, according to classical logic, individuals should use. Nevertheless, they only apply it in certain cases. But, as pointed out above and argued in a detailed way in papers such as that of Orenes and Johnson-Laird (2012), the mental models theory has the machinery necessary to account for facts such as these. Under its framework, it is evident why [III] cannot be derived from a premise such as ‘Pat is here’. The reason is [IV.2], that is, a possibility that can be attributed to [III] and in which the premise ‘Pat is here’ is false. So, the idea is that, from a sentence, it cannot be inferred another sentence that is consistent with a situation in which the former is false. Clearly, on the contrary, this difficulty does not exist in the hypothetical case in which the premise refers to the circumstance of Lucia wearing jewelry and the conclusion is [VII]. And it does not exist in that case because there are no any possibilities that can be linked to [VII] in which Lucia cannot wear jewelry (explanations akin to this one are given in several works as well, e.g., López-Astorga, 2017).

Nonetheless, beyond examples such as this one, what is truly important for this paper is that, following what in the theory is deemed as necessary and possible, there are significant differences between, on the one hand, [I] and [V], and, on the other hand, [III] and [VII]. Indeed, according to Khemlani, Hinterecker, and Johnson-Laird [2017], under the framework of the mental models theory, an element is possible when it happens in, at a minimum, one of the possibilities corresponding to its sentence. In the same way, an element is necessary when it happens in all of the possibilities corresponding to its sentence. But, if this is so, in [I], as shown by [II], both [he promised to take the kids to the zoo] and [he takes the kids to the zoo] are possible, and, in [III], as shown by [IV], both

[Pat is here] and [Viv is here] are possible. However, in [V], as shown by [VI], while [your apples fall] continues to be possible, [there is gravity] is necessary, and, in [VII], as shown by [VIII], while [Lucia wore the bracelet] keeps being possible, [Lucia wore jewelry] is necessary.

As explained below, these definitions of necessity and possibility satisfy the relationships provided by the Aristotelian modal square of opposition, which means, following Fitting and Mendelsohn [1998], that they fulfill the requirement necessary to build a modal logic. Nevertheless, this in turn also reveals that, while the mental models theory seems to be right that the human mind is not directly related to classical logic, it can be a mistake to deduce from that fact that it is related to no logic of any kind. The links between the definitions of necessity and possibility above and the Aristotelian square suggest that, on the contrary, there can be a logic leading human reasoning and hence a syntax of thought, even if that syntax is not such as the one described by, for example, Braine and O'Brien [1998], but akin to the one proposed in papers such as, for instance, that of López-Astorga [2017], that is, a syntax with characteristics of modal logics. But, to check all of this, maybe it is necessary firstly to describe the modal square of opposition in detail.

The Aristotelian Modal Square of Opposition

It is really well known that the relationships of opposition are very important in Aristotelian logic. Nonetheless, as indicated, for example, by Pabijutaitė [2018], only the part of it deemed as 'basic' is not considered to be problematic, the modal part having caused difficulties from its very beginning. Perhaps, for this last circumstance, it is better to start with the 'basic' square, and then to move forward to the modal one.

Actually, the 'basic' square has already been analyzed from the perspective of the mental models theory [López-Astorga, 2016]. However, the aims and the intentions of that previous analysis were very different from those in this paper, as, in it, the mental models theory was essentially used for trying to predict what Aristotelian relationships of opposition should be hard to explain to students in a philosophy class. Nevertheless, maybe the simplest option is to do as in that analysis and to resort to the description of the square of opposition that appears in *Tractatus* (*Summulae Logicales*) by Peter of Spain.

As it is also known, the assertoric square is based on four kinds of quantified sentences, which, from the words *AffIrmo* and *nEgO*, are habitually denominated 'A', 'E', 'I', and 'O'. A and E are universally quantified, the difference between them being that E is negated. On the other hand, I and O are particularly quantified, the difference between them being, likewise, that O is negated.

In this way, the laws regulating the relationships are the following:

- Law of contraries [*contrariarum lex*: Peter of Spain, *Tractatus* I, 12F. See also, e.g., López-Astorga, 2016: 36. Fitting; Mendelsohn, 1998: 7]: it provides that, if A is true, E has to be false, and, if E is true, A has to be false. Indeed, if A means 'every S is P' and E denotes 'no S is P', it is clearly obvious that, if one of them is true, the other one must be false.

- Law of subcontraries [*subcontrariarum lex*: Peter of Spain, *Tractatus* I, 13B. See also, e.g., López-Astorga, 2016: 36. Fitting; Mendelsohn, 1998: 7]: it provides that, if I is false, O has to be true, and, if O is false, I has to be true. This law is also obvious. Certainly, if I refers to 'some S is P' and O stands for 'some S is not P', it is not possible that they are false at the same time, since, if it were so, S could be neither P nor not-P, and any S must be either P or not-P.

- Law of contradictories [*contradictoriarum lex*: Peter of Spain, *Tractatus* I, 13B. See also, e.g., López-Astorga, 2016: 36. Fitting; Mendelsohn, 1998: 8]: it provides that the truth-value of A has to be different from the one of O, and that the truth-value of E has to be different from the one of I. And this law is easy to understand too. If every S is P (A is true), then no S is not P (O is false), and if it is not true that every S is P (A is false), then, at least, some S is not P (O is true). In the same way, if some S is not P (O is true), then it is not possible that every S is P (A is false), and, if it is not true that some S is not P (O is false), then that means that every S is P (A is true). On the other hand, if no S is P (E is true), then it is not possible that some S is P (I is false), and, if it is not true that no S is P (E is false), then it must be accepted that, at least, some S is P (I is true). In the same way, if some S is P (I is true), it is impossible that no S is P (E is false), and, if it is not true that some S is P (I is false), then, necessarily, no S is P (E is true).

- Law of subalternations [*subalternarum lex*: Peter of Spain, *Tractatus* I, 13F. See also, e.g., López-Astorga, 2016: 36. Fitting; Mendelsohn, 1998: 8]: it provides that, if A is true, I is so as well, and, if E is true, O is so as well. Evidently, if every S is P (A is true), it can also be said that some S is P (I is true), and, if no S is P (E is true), it can also be said that some S is not P (O is true).

Those are the laws ruling the relationships between A, E, I, and O in the assertoric square. However, paying attention to the Aristotelian work Περὶ Ἑρμηνείας (*De Interpretatione*), and, in particular, to its chapters 12 and 13, Fitting and Mendelsohn [1998] raise the usual definitions of the alethic qualifiers in modal logic, that is, the following definitions (see Fitting; Mendelsohn, 1998: 7]:

[IX]

1. $\Diamond p \longleftrightarrow \neg \Box \neg p$
2. $\Box p \longleftrightarrow \neg \Diamond \neg p$

(Where '$\Diamond$' is the operator of possibility, '$\Box$' is the one of necessity, '$\longleftrightarrow$' expresses biconditional relationship, and '$\neg$' denotes negation).

And they remind that, with the definitions in [IX], it is possible to build a modal logic fulfilling the four laws above too. This is not hard to see if, as it is done by Fitting and Mendelsohn [1998: 7, Figure 1], these equivalences are assumed:

[X]

1. A = □p or ¬◊¬p.
2. E = □¬p or ¬◊p.
3. I = ¬□¬p or ◊p.
4. O = ¬□p or ◊¬p.

Starting by the law of contraries, it can be claimed that, indeed, □p and □¬p cannot be true at the same time. If it were so, for being necessary, both p and ¬p would be always true at the same time, which would always lead one to an inconsistency. And this apart from the fact that, as indicated by Fitting and Mendelsohn [1998], in many modal logics, a formula such as the following generally holds:

[XI](□p ∧ □q) →□(p ∧ q)

(Where'∧' represents conjunction and'→' is the conditional).

Which, if □p and □¬p are true, leads to the contradiction:

[XII]□(p ∧ ¬p)

Likewise, as pointed out by the law of subcontraries, ¬□¬p and ¬□p cannot be false at the same time. If that were so, both □¬p and □p would be true, which would lead to a contradiction, or, if preferred, to [XII], again.

As far as the law of contradictories is concerned, the possible situations are also clear. If □p and ¬□p were both true, the inconsistency would be, once again, obvious, as well as if they were both false, as, in that case, ¬□p and □p would be true too. In the same way, the contradiction would be also evident if □¬p and ¬□¬p were the formulae true at the same time, or the formulae false at the same time, since, in this last case, again, the true formulae would be ¬□¬p and □¬p.

Finally, regarding the law of subalternations, it cannot be accepted that □p is true and ◊p is false, because then □p and □¬p would be true again, which would cause an incoherency, and, by means of [XI], could also lead to [XII]. In the same manner, if □¬p and □p (i.e., the negation of ¬□p) were the true formulae, the situation would be clearly identical.

So, there is no doubt that, from the formulae in [IX], a modal logic satisfying, as required in works such as that of Fitting and Mendelsohn [1998], the Aristotelian square of opposition can be constructed. This is obvious and it is not the point of this paper. The point of this paper is that the concepts of necessity and possibility as defined by the mental models theory meet that requirement as well. Of course, one might think that, given the previous accounts, this is also evident. However, because the mental models theory tends to reject any kind of logic and syntax, it can be worth developing that idea, and this is exactly what is done in the next section.

The Relationships of Opposition and the Definitions of Necessity and Possibility in the Mental Models Theory

To check that the concepts of necessity and possibility as understood by the mental models theory, certainly, allows building a modal logic fulfilling the laws corresponding to the square of opposition, it can be enough to consider these equivalences:

[XIII] A = the particular element is necessary, that is, happens in all the possibilities linked to the particular sentence in natural language.

[XIV] E = the denial of the particular element is necessary, that is, happens in all the possibilities linked to the particular sentence in natural language.

[XV] I = the particular element is possible, that is, happens, at a minimum, in one of the possibilities linked to the particular sentence in natural language.

[XVI] O = the denial of the particular element is possible, that is, happens, at a minimum, in one of the possibilities linked to the particular sentence in natural language.

Indeed, given these equivalences, it is not difficult to note that the four laws dealt with in the last section are satisfied in this case too. With regard to the law of contraries, it is obvious that [XIII] and [XIV] are not situations that can occur at the same time. A particular element and its denial cannot be both true in all the possibilities corresponding to a particular sentence in natural language. If they were so, both the element and its denial would be in each of such possibilities, which cannot be admitted by the mental models theory. As point out, this last theory considers just the different viable, feasible, and consistent possibilities that can be attributed to sentences, and, undoubtedly, a possibility including an element and its negation is neither viable, nor feasible, nor consistent.

On the other hand, if the law of subcontraries is addressed, it must be stated that, truly, either [XV] or [XVI], or both, have to be true. [XV] provides that a particular element is possible and [XVI] claims that the denial of that very element is possible. But, for exactly this reason, they cannot be false at the same time. If [XV] is false, the element is not possible, and that means that it cannot appear in any possibility, which transforms the denial of that element into necessary, since it has to be in all the possibilities. However, if the denial is necessary, then it is also possible, as, if it is in all the possibilities of a sentence, it is also, at a minimum, in one of them, which makes [XVI] true. Likewise, something similar can be said regarding the case in which [XVI] is false. If it is so, that means that the element has to appear in all the possibilities, hence getting necessary. Nevertheless, if the element is necessary, then it is also possible, because, if it is in all the possibilities linked to a sentence, it is, at a minimum, in one of them too, which makes [XV] true.

As it can be drawn from what has been said, the law of contradictories claims in this case that, on the one hand, [XIII] and [XVI], and, on the other hand, [XIV] and [XV] cannot have the same truth-value. And this is absolutely correct as well here. If [XIII] is true, the element is necessary and its denial cannot be in any possibility, that is, is not possible, which means that [XVI] is false. In the same way, if [XIII] is false, that implies that the denial of the element must appear in, at a minimum, one possibility of the sentence, which is exactly what is provided by [XVI]. In a similar manner, if [XVI] is false, the denial of the element can happen in no possibility, that is, the element is necessary and, therefore, [XIII] is true. And if [XVI] is true, it is not true that the element occurs in all the possibilities linked to the sentence, that is, that the element is necessary, which reveals that [XIII] is not true.

Likewise, if [XIV] is true, the denial of the element is in all the possibilities, and, for this reason, the element cannot appear in any of them, which makes [XV] false. In the same manner, if [XIV] is false, the denial of the element is not necessary and the element happens, at a minimum, in one of the possibilities, which is precisely what is indicated by [XV]. In a similar way, if [XV] is true, [XIV] cannot be so, since there is, at a minimum, a possibility in which the element is. And, of course, if [XV] is false, [XIV] needs to be true, since, if there is no possibility in which the element occurs, its denial is necessary and [XIV] is absolutely true.

Furthermore, as far as the law of subalternations is concerned, it is obvious that, if, as provided by [XIII], an element is necessary, it appears in all the possibilities related to the sentence. However, if it appears in all of those possibilities, it is also true that it is, at a minimum, in one of them, which shows that [XV] is true as well. Likewise, if [XIV] is the case that is true, then the denial of the element is necessary, which leads to the fact that that denial is present in all the possibilities. Nevertheless, as in the previous case, if that denial is present in all the possibilities of the sentence, then it can also be stated that it happens, at a minimum, in one of them, which in turn reveals that [XVI] is true too.

Accordingly, the concepts of necessity and possibility interpreted under the framework of the mental models theory satisfy the modal version of the square as well. So, following what Fitting and Mendelsohn [1998] indicate, it is hard to ignore the possibility that it really refers to a deep underlying modal logical system of some type.

Conclusions

Certainly, the mental models theory fulfills the basic requirement to be a clear modal logic. Therefore, it can also be thought that, at the very least, a modal system can be constructed from its essential theses. Undoubtedly, formal relationships between its concepts can be found. Thus, if its general approach is correct, and, following the specialized literature, one might think that that is really so [see, e.g., López-Astorga, 2017], it seems that it is still possible to keep speaking about a syntax of thought.

Of course, that syntax would not be such as the one proposed by the mental logic theory. In fact, it does not need to be even, as raised by López-Astorga [2017], very akin to system K, although what has been argued above gives reasons to think that this last idea is a reasonable option too. However, if something is beyond doubt, that is that, in line with approaches such as, for instance, the one of López-Astorga [2017], the mental models theory does not eliminate the possibility that some kind of logic linked to language, communication, and the inferential activity is present in the human mind.

Hence it appears that what is most appropriate from now on is, following arguments such as those of López-Astorga as well, to continue to explore which that logic is exactly. The analyses above and the ones offered in papers such as that of López-Astorga [2017] suggest that a logic of that type should be basically modal, but, because, as it is well known, there are several modal logics, what is really relevant can be to attempt to determine the real features of it. Maybe, as said, it is similar to K. Nevertheless, the possibility that further reviews lead to qualify the characteristics of the real modal syntactic system linked to human thought, and, for example, to add to it some more axiom and, in this way, to transform it into other modal system already known, or even into another completely different one, should be always taken into account too.

From this point of view, paying attention, once again, to works such as, for example, the one of López-Astorga [2017], it can be said that what should be done is not to reject the mental models theory, but, assuming its general framework, to revitalize syntax in it. And this is so because it can be raised that the mental models theory has both a strength and a weakness. Its strength is that it highlights the actual importance that semantics has. Its weakness is that it forgets or relegates syntax to a second place.

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