

SOME DEONTIC LOGICAL FORMULAE THAT CAN BE USED IN A NATURAL WAY IN HUMAN REASONING

Miguel López-Astorga – PhD in PhD in Logic and Philosophy of Science, associate professor. Instituto de Estudios Humanísticos “Juan Ignacio Molina”, University of Talca. Av. Lircay s/n, 3460000 Talca, Chile;
e-mail: milopez@utalca.cl



The mental models theory proposes that reasoning is not logical. From its point of view, people only regard possibilities that represent reality in an iconic way, and they draw conclusions by reviewing such possibilities. Obviously, a framework of this kind seems incompatible with the idea that the human mind thinks by means of logical forms. However, in this paper, the author tries to show that, if we accept the basic theses of the mental models theory, we must also assume that certain formal logical structures are equally part of the human intellectual machinery, even though its proponents explicitly reject any link between logic and thought. In particular, the author argues here that it is not possible to adopt the mental models theory without accepting, at the same time, some deontic propositions that are usually admitted in standard deontic logic.

Keywords: deontic logic, iconic possibilities, logical form, mental models, reasoning

НЕКОТОРЫЕ ДЕОНТИЧЕСКИЕ ЛОГИЧЕСКИЕ ФОРМУЛЫ И ИХ НЕПОСРЕДСТВЕННОЕ ИСПОЛЬЗОВАНИЕ В ЧЕЛОВЕЧЕСКОМ МЫШЛЕНИИ

Лопез-Асторга Мигель – доктор философии, доцент. Институт гуманитарных исследований им. Хуана Игнасио Молина, Университет Талька. Av. Lircay s/n, 3460000, Талька, Чили;
e-mail: milopez@utalca.cl

Теория ментальных моделей утверждает, что мышление не является логическим. С этой точки зрения, реальность воспринимается людьми благодаря знаковым возможностям, анализ которых позволяет людям делать о ней выводы. Очевидно, что такого рода идея представляется несовместимой с предположением о том, что человек мыслит логическими формами. Тем не менее, в этой статье автор стремится показать, что если мы принимаем положения теории ментальных моделей, то должны принять и то, что определенные логические структуры также являются частью мыслительного процесса. Пусть с этим и не согласятся сторонники названной теории, которые отрицают всякую связь между логикой и мыслью. Автор этой статьи полагает, что невозможно согласиться с теорией ментальных моделей без принятия некоторых положений деонтической логики.

Ключевые слова: деонтическая логика, знаковые возможности, логическая форма, ментальные модели, мышление



Introduction

An important theory is nowadays the mental models theory, or, as called in Quelhas, Rasga, and Johnson-Laird [2017], ‘the unified theory of the mental models’. This framework is relevant today because the experimental results usually obtained appear to support its main theses [vide, e.g., Johnson-Laird; Khemlani; Goodwin, 2015a]. However, what is interesting about it for this paper is one of its basic assumptions. According to the theory, individuals do not reason in a logical way, but by combining different iconic possibilities describing reality that they build from what is transmitted by the sentences in natural language [vide, e.g., Johnson-Laird; Girotto; Legrenzi, 1999]. Thus, adopting a concept of iconicity akin to that of Peirce [1931–1958], this approach claims that the possibilities fairly comprehensively represent alternative scenarios, and that the inferences are made by joint consideration of such possibilities, general knowledge, and the global context in which the sentences are interpreted [vide, e.g., Johnson-Laird, 2012].

Obviously, a theory of this type is *prima facie* inconsistent with the thesis that people identify the logical forms referred by the sentences, and that they make deductions by using formal schemata whose premises match those logical forms. In fact, its main proponents often clearly decline ideas in this direction [vide, e.g., Johnson-Laird, 2010]. Nevertheless, I will intend to show in this paper that the mental models theory sometimes leads to consequences unexpected by its adherents, and that the cognitive machinery that it assigns to the human mind can cause us to assume that we resort to a logic of any kind when making inferences. In this way, my essential idea will be similar to a general thesis developed in several papers linking the possibilities of the mental models theory to different kinds of logic, merely one of them being, for example, that of López-Astorga [2015]. That general thesis is that there are clear relations between the iconic possibilities of this last theory and syntactic forms, and that hence the findings of the mental models theory can reveal just the opposite of what it wishes to demonstrate, i.e., that there are formal structures in our mind.

In particular, this paper is aimed to argue that some iconic possibilities that can be considered in the deontic context by a human being are absolutely compatible with the formal structure of certain propositions habitually accepted in standard deontic logic. The basis of the argument will be that those possibilities and those propositions share the same sense, and that, therefore, the possibilities exactly refer to the same logical relationships as the propositions.

So, to do all of the foregoing, I will begin with a description of the general framework of the mental models theory. Then, I will indicate which the deontic formulae that can be related to iconic possibilities of this last



theory are and what they provide. Finally, I will show how, based on the fundamental assumptions of the mental models theory, it can be predicted that an average individual will deem such formulae to be valid. Hence the first section is about the mental models theory.

The mental models theory and the importance of semantics and pragmatics

As said, a key concept in the mental models theory is the one of iconic possibility or representation. In this way, it can be said that the alternative scenarios described by the possibilities or representations for a same proposition are, in general, very similar and only have minimal variations. An example in this regard can be that of the conditional. Following the theory, conditionals, that is, the propositions with the structure ‘if p, then q’, refer to three alternative scenarios [vide, e.g., Quelhas et al., 20176, p. 1006–1007]:

[I]:	p	q
[II]:	not-p	q
[III]:	not-p	not-q

The three worlds represented by [I], [II], and [III] are almost identical and their differences are minor. World [I] is different from world [II] only because in [I] p occurs and in [II] it does not. [II], in turn, is different from [III] only because in [II] q happens and in [III] it does not. Therefore, as mentioned, the differences between the possibilities are never very major.

But, if the foundation of the mental models theory were only theses such as the previous one, one might think that, actually, it is akin to standard logic. Indeed, the truth table of the conditional in this last logic provides that a formula such as $p \rightarrow q$ (where ‘ \rightarrow ’ stands for the logical conditional) is valid in three cases that appear to match scenarios [I], [II], and [III]. And this is so because, adopting symbols similar to, for example, those used in López-Astorga [2015], it can be stated that

$$v(p \rightarrow q) = 1 \text{ IFF } v[(p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q)] = 1$$

Where ‘ $v(x)$ ’ denotes ‘truth value of x’, ‘1’ represents truth, ‘IFF’ is an abbreviation of ‘if and only if’, ‘ \wedge ’ refers to conjunction, ‘ \vee ’ is the symbol for disjunction, and ‘ \neg ’ corresponds to negation.

And that

$$v(p \rightarrow q) = 0 \text{ IFF } v(p \wedge \neg q) = 1$$

Where ‘0’ means ‘false’.



Evidently, the formula $(p \wedge q) \vee (\neg p \wedge q) \vee (\neg p \wedge \neg q)$ seems to refer to the same situations represented by [I], [II], and [III]. Besides, it appears that $p \wedge \neg q$ can be linked to the only world that, according to the mental models theory, conditionals do not admit in principle, that is, to a scenario such as this one:

[IV]: $p \quad \text{not-}q$

Nevertheless, as indicated, for example, in López-Astorga [2016], this is not so at all. On the one hand, as also explained here, because the possibilities iconically represent reality, p and q are not logical variables in them, but just iconic elements. On the other hand, the theory equally claims that people are not always able to think about all the possibilities that can be related to a sentence, which, in the particular case of the conditional, means that they are not always able to think about [I], [II], and [III], and that, in many occasions, they can only consider a possibility such as [I] [vide, e.g., Quelhas et al., 2017, p. 1006]. Furthermore, the mental models theory proposes mental processes of modulation as well [vide, e.g., Johnson-Laird et al., 2015a, p. 202]. Those processes are caused by contextual, semantic, and pragmatic factors, and produce as a result combinations of possibilities in which certain scenarios are eliminated or modified.

This last thesis can be better understood by means of an example. Let us think about this conditional sentence:

“If it is raining, then it is pouring” [Quelhas et al., 2017, p. 1007].

Undoubtedly, this is a conditional sentence, but, if we consider p to be equivalent to ‘it is raining’ and q to be equivalent to ‘it is pouring’, it is very hard to assign [II] to it, since, as it is known, the fact that it is pouring is not possible without the fact that it is raining. Nonetheless, the meaning the ‘pouring’ could lead to a world such as that represented in [IV], as it can be raining without pouring [Quelhas et al., 2017, p. 1007ff].

So, it is clear that the machinery of the mental models theory is very different from that of standard logic. As just shown, the former can block situations correct following the latter, as well as conclusions that are not allowed in the latter can be possible in the former. However, none of this has an influence on certain basic structures of standard deontic logic, which can lead to think that there is something akin to a deontic mental logic characterized by essentially formal and syntactic structures. The reason why this can be so will be explained below. Nevertheless, before arguing in favor of the idea that, if theses such as those of the mental models theory commented on in this section are assumed, such structures must be accepted too, it appears to be suitable to indicate which those formal structures are exactly.



Some propositions habitual in standard deontic logic

Usually, the expression deontic logic is linked to works such as the ones authored by Von Wright [1951, 1956, 1963]. Nonetheless, it is well known that all the deontic logics are not the same and that hence not all of them share the same assumptions. Despite this, Forrester [1996] points out nine propositions that are often assumed in standard deontic logic. But, before introducing them, it seems to be necessary to remind the definitions of the two operators that are not included in general standard logic and that are habitually used in standard deontic logic. Such operators are, obviously, the one of obligation (O) and the one of permission (P), and they are generally defined by each other in this way:

$$\begin{aligned} \text{[V]:} & \quad O_x =_{df} \neg P\neg x \\ \text{[VI]:} & \quad P_x =_{df} \neg O\neg x \end{aligned}$$

Where, clearly, [V] means that ‘x is obligatory’ is equal to ‘not-x is not permitted’, and [VI] means that ‘x is permitted’ is equal to ‘not-x is not obligatory’.

That said, the propositions [which are to be found in Forrester, 1996, p. 26–27] can be presented now. They are the following:

$$\begin{aligned} \text{[VII]:} & \quad \neg(O_p \wedge O\neg p) \\ \text{[VIII]:} & \quad O(p \rightarrow q) \rightarrow (O_p \rightarrow Oq) \\ \text{[IX]:} & \quad O(p \wedge q) \rightarrow (O_p \wedge Oq) \\ \text{[X]:} & \quad (O_p \wedge Oq) \rightarrow O(p \wedge q) \\ \text{[XI]:} & \quad \neg O(p \wedge \neg p) \\ \text{[XII]:} & \quad O(p \vee \neg p) \\ \text{[XIII]:} & \quad O_p \leftrightarrow \neg P\neg p \\ \text{[XIV]:} & \quad P_p \leftrightarrow \neg O\neg p \\ \text{[XV]:} & \quad O_p \rightarrow P_p \end{aligned}$$

Where ‘ \leftrightarrow ’ stands for biconditional relationship.

Apart from the fact that the names these formulae receive in Forrester [1996, p. 26–27] are different, and not just Roman numbers VII to XV, two of my symbols are not those of Forrester, who uses ‘ \sim ’ for negation and ‘ \cdot ’ for conjunction. However, maybe a more important point here is that [XIII] and [XIV] correspond to definitions [V] and [VI]. And this is relevant because, if it is shown, as this paper intends, that, if we consider the mental models theory to be a correct theory, we must also consider formulae [VII] to [XV] to be common sense formal propositions, it will also be shown that, under the framework of this last theory, the definitions of the operators ‘O’ and ‘P’ in standard deontic logic are natural for the human mind and syntactic structures of our thought as well. So, it can be said that, beyond Forrester’s actual goals, the next section tries to argue in this direction.



Some deontic formulae and its iconic possibilities

That a proponent of the mental models theory would have to admit [VII] is something that can be easily checked if we simply ignore its main negation and pay attention to the content between brackets. If we do that, we can observe that that content refers to a scenario in which an action and its contrary are mandatory at the same time, that is, to a scenario akin to this one:

[XVI]: p is obligatory not- p is obligatory

This is so because, according to the mental models theory, conjunction a priori only refers to one iconic representation such as [I] [vide, e.g., Johnson-Laird, 2012, p. 138, Table 9.2]. But, given that, based on the references on this last theory cited above, an iconic representation such as [XVI] cannot be taken as a possibility (it is almost impossible to admit that a representation describing an action and its denial as mandatory shows a plausible scenario), there is no doubt that the basic theses of the mental models theory lead to reject formal structures such as $Op \wedge O\neg p$, and hence to accept formulae such as $\neg(Op \wedge O\neg p)$.

As regards [VIII], we can focus on the antecedent of the conditional, that is, on $O(p \rightarrow q)$. Following the mental models theory, what this last formula provides is that it is absolutely obligatory that one of the three possibilities [I], [II], and [III] happens. But, if this is so, we have that q occurs in the only case in which p occurs too ([I]). Therefore, if p is obligatory, [I] reveals that q must be true, which means that, if p is obligatory, then q is also so, i.e., that $Op \rightarrow Oq$. Thus, if $O(p \rightarrow q)$ is admitted, $Op \rightarrow Oq$ has to be admitted as well, and hence $O(p \rightarrow q) \rightarrow (Op \rightarrow Oq)$ has to be admitted too.

In connection with [IX], $O(p \wedge q)$ clearly only allows a scenario in which it is mandatory that p and q happen at the same time. Between the brackets, there is a conjunction, and, as pointed out, conjunctions generally lead to [I] in the mental models theory. However, if [I] is obligatory, p and q are both of them obligatory too, and it can be stated that p is obligatory (Op) and that q is obligatory (Oq). Therefore, if $O(p \wedge q)$ is true, Op and Oq , i.e., $Op \wedge Oq$ is so as well, which in turn shows that $O(p \wedge q) \rightarrow (Op \wedge Oq)$ must be assumed.

As far as [X] is concerned, the situation is very similar. Because $Op \wedge Oq$ is a conjunction, it only enables this iconic possibility:

[XVII]: p is obligatory q is obligatory

Nevertheless, it is evident that [XVII] iconically represents a situation in which [I] is mandatory, and hence a scenario in which it is mandatory that p and q happen at the same time, which is an idea that can be well expressed by means of the formula $O(p \wedge q)$. So, it is correct that $(Op \wedge Oq) \rightarrow O(p \wedge q)$.



The case of [XI] is akin to that of [VII]. The conjunction $p \wedge \neg p$ can only stand for an impossible situation in the mental models theory, and, for this reason, it cannot be linked to a status quo that is obligatory. In other terms, it cannot be proposed that $O(p \wedge \neg p)$, and, accordingly, only $\neg O(p \wedge \neg p)$ seems to make sense.

In [XII], we find a disjunction, and the mental models theory also attributes a priori iconic representations to this last connective, both in the case in which it is inclusive and in the case in which it is exclusive. Undoubtedly, from what has been explained above, it is evident that a disjunction such as $p \vee \neg p$ cannot be inclusive in the mental models theory, since it would imply a scenario in which p and $\neg p$ are true at the same time [for a discussion about the actual way the mental models theory deals with disjunctions such as $p \vee \neg p$, vide, e.g., Johnson-Laird; Khemlani; Goodwin, 2015b, p. 549]. Thus, the disjunction in [XII] must be exclusive, and, following Johnson-Laird [2012, p. 138, Table 9.2], an exclusive disjunction such as ‘ p or q but not both of them’ can be related to two possibilities: [II] and [IV], which, in the case of $p \vee \neg p$, can be translated into:

[XVIII]:	p	not-not- p
[XIX]:	not- p	not- p

Clearly, [XVIII] refers to a world in which p is true and $\neg p$ is not, and [XIX] to an inverse world, one in which $\neg p$ is true and p is not. However, the important point here is that, although the mental models theory tries to ignore logic, it does not entirely achieve that. *Principium Tertii Exclusi* seems to hold in it and one of these two last scenarios must be the real one. This is so because, in the same way as the two disjuncts cannot be true at the same time, they cannot be false at the same time either (since, obviously, in this last circumstance, we would have a scenario with p and $\neg p$ again). Accordingly, there are only two options: i) p happens or ii) p does not happen. If i) is the case, [XVIII] is the actual world. On the contrary, if ii) is the case, the real world is [XIX]. But, as said, one of them must be the actual scenario, that is, it is obligatory $p \vee \neg p$, or, if preferred, $O(p \vee \neg p)$.

[XIII], which, as said, corresponds to [V], includes another connective that is also taken into account by the mental models theory: the biconditional. According to the theory, the possible scenarios that can usually be linked to this last connective are [I] and [III] [vide, e.g., Johnson-Laird, 2012, p. 138, Table 9.2], which leads one to think that the possibilities of [XIII] are:

[XX]:	p is obligatory	not- p is not permitted
[XXI]:	p is not obligatory	not- p is permitted

However, given that these possibilities are iconic representations and it is evident that in [XX] ‘ p is obligatory’ and ‘not- p is not permitted’ have the same meaning, [XX] can be considered to be a particular case of



[XVIII]. In the same way and for similar reasons, [XXI] can be thought to be a particular instance of [XIX]. But, if this is so and, as explained above, [XVIII] and [XIX] cause $O(p \vee \neg p)$ to be accepted, it can be argued that $O_p \leftrightarrow \neg P\neg p$ is just a particular case of $O(p \vee \neg p)$ and that hence it must be assumed for the same motives as this last formula.

The account is not very different in the case of [XIV]. Because, again, the main symbol of the formula is a biconditional, its scenarios are as follows:

[XXII]:	p is permitted	not-p is not obligatory
[XXIII]:	p is not permitted	not p is obligatory

As in [XIII], it can be assumed that the two elements of each scenario have the same sense, since ‘p is permitted’ is iconically equivalent to ‘not-p is not obligatory’, and ‘p is not permitted’ is iconically equivalent to ‘not-p is obligatory’ too. In this way, it can be claimed that [XXII] and [XXIII] are again particular examples of, respectively, [XVIII] and [XIX], and that, in the same manner as these two last possibilities justify the acceptance of [XII], [XXII] and [XXIII] allow supposing [XIV].

Finally, the explanation for [XV] is very easy. If O_p is true, then we have a possible world such as [XX]. In this last world, p is mandatory and hence it must happen. Thus, given that that world is, as explained, from the framework of the mental models theory, a iconic description of reality, in it, it is not possible for p not to be permitted, since, if that were so, p should not occur, and it is, as said, obligatory. And, obviously, in an iconic scenario in which it is not possible for p not to be permitted, p is undoubtedly permitted (P_p). So, if p is obligatory, then p is permitted, or, if preferred, $O_p \rightarrow P_p$.

Accordingly, it is absolutely clear that, if essential theses of the mental models theory such as those commented on above are admitted, it is necessary to accept formulae [VII] to [XV] too. And this is so in spite that the real proposal of this last theory is to ignore logical forms.

Conclusions

But, if this is correct and there are formal structures that people can admit in an easy and natural way, it seems that, beyond its proponents’ actual intentions, the acceptance of the mental models theory also implies the possibility to speak about a mental logic in human beings. Really, there are several theories in this regard, for example, the one of Henlé [1962], the one of Rips [1994], or the one of the so-called ‘mental logic theory’ [vide, e.g., Braine; O’Brien, 1998; Gouveia; Roazzi; Moutinho; Bompastor Borges Dias; O’Brien, 2002; O’Brien, 2014; O’Brien; Li, 2013]. In fact,



there are even approaches that claim the existence of a special logic in our mind that is applied only in deontic contexts [vide, e.g., Cheng; Holyoak, 1985; 1989; Fodor, 2000]. However, as far as I know, what is new in this paper is that it shows that, if we reject all these theories and we choose the mental models theory, we still have to assume certain basic formulae habitual in standard deontic logic.

Evidently, this does not demonstrate at all that the mental models theory does not hold. It has strong empirical evidence in its favor, and that fact has to be acknowledged. Nevertheless, my arguments here do reveal that it is very possible for the mental models theory to be closer to the formal or syntactic theories such as those that propose a logic for the human mind than thought. Thus, it can be argued, as, for example, López-Astorga [2015] does, that the mental models theory and the syntactic proposals complement each other, since they address two different dimensions of language and the intellectual activity.

Nonetheless, the mental models theory deserves that it is also recognized that it will always have, in any case, certain priority, this last point being one of those that are highlighted in studies such as the one of López-Astorga [2015, p. 148] as well. Indeed, according to López-Astorga, the literature on human reasoning proves that the iconic possibilities of the mental models theory alone can predict the answers that will be offered by participants in many reasoning tasks without the need to consider logical formulae. However, this is not right the other way round, since it is not possible even to account for some of those very results using only logical formulae. In addition, as López-Astorga's main argument makes it evident, in many times, to identify logical forms, a prior review of the iconic possibilities of the sentences is *conditio sine qua non*. And this last fact can also show that the theses of the mental models theory can have a temporal priority too, as it can be necessary a previous recourse to the machinery of the mental models theory to get logical forms later, i.e., to get the elements that, following the syntactic theories, our mind needs to work later.

But this paper appears to reveal that, while it can be true that we cannot forget the basic assumptions of the mental models theory and that it is always required to take them into account, there may be also no doubt that, if it is not impossible, it is at a minimum very difficult to present a cognitive theory removing the fundamental elements of standard deontic logic. Of course, the iconic representations can be indispensable in the human intellectual activity, but it does not mean that, at least to some extent, certain formal elements are not so at the same time. In this way, it seems that it is worth continuing to research following this line, since it can help us identify, with as little mistakes as possible, which such formal elements are exactly. As it can be checked above, my main intention here has been precisely this last one.



Список литературы

- Braine; O'Brien, 1998 – *Braine M.D.S., O'Brien D.P.* (eds.). *Mental Logic*. Mahwah, NJ: Lawrence Erlbaum Associates, Inc., Publishers, 1998. 481 p.
- Cheng, Holyoak, 1985 – *Cheng P.W., Holyoak K.J.* Pragmatic reasoning schemas // *Cognitive Psychology*. 1985. No. 17. P. 391–416.
- Cheng, Holyoak, 1989 – *Cheng P.W., Holyoak K.J.* On the natural selection of reasoning theories // *Cognition*. 1989. No. 33. P. 285–313.
- Fodor, 2000 – *Fodor J.* Why are so good at catching cheaters // *Cognition*. 2000. No. 75. P. 29–32.
- Forrester, 1996 – *Forrester J.W.* *Being Good and Being Logical: Philosophical Groundwork for a New Deontic Logic*. N. Y.: M. E. Sharpe, Inc, 1996. 332 p.
- Gouveia et al, 2002 – *Gouveia E.L., Roazzi A., Moutinho K., Bompastor Borges Dias M.G., O'Brien D.P.* Raciocínio condicional: influências pragmáticas // *Estudos de Psicologia*. 2002. Vol. 7. No. 2. P. 217–225.
- Henlé, 1962 – *Henlé M.* On the relation between logic and thinking // *Psychological Review*. 1962. No. 69. P. 366–378.
- Johnson-Laird, 2010 – *Johnson-Laird P.N.* Against logical form // *Psychologica Belgica*. 2010. Vol. 5. No. 3/4. P. 193–221.
- Johnson-Laird, 2012 – *Johnson-Laird P.N.* Inference with mental models // *The Oxford Handbook of Thinking and Reasoning* / Ed. by K.J. Holyoak, R.G. Morrison. N. Y.: Oxford University Press, 2012. P. 134–145.
- Johnson-Laird et al., 1999 – *Johnson-Laird P.N., Girotto V., Legrenzi P.* Modelli mentali: Una guida facile per il profano // *Sistemi Intelligenti*. 1999. Vol. XI. No. 1. P. 63–84.
- Johnson-Laird et al., 2015a – *Johnson-Laird P.N., Khemlani S., Goodwin G.P.* Logic, probability, and human reasoning // *Trends in Cognitive Sciences*. 2015. Vol. 19. No. 4. P. 201–214.
- Johnson-Laird et al., 2015b – *Johnson-Laird P.N., Khemlani S., Goodwin G.P.* Response to Baratgin et al.: Mental models integrate probability and deduction // *Trends in Cognitive Science*. 2015. Vol. 19. No. 10. P. 548–549.
- López-Astorga, 2015 – *López-Astorga M.* The disjunction introduction rule: Syntactic and semantic considerations // *Pragmalingüística*. 2015. No. 23. P. 141–149.
- López-Astorga, 2016 – *López-Astorga M.* The first rule of Stoic logic and its relationship with the *indemonstrables*. // *Tópicos. Revista de Filosofía*. 2016. No. 50. P. 9–23.
- O'Brien; Li, 2013 – *O'Brien D.P., Li S.* Mental logic theory: A paradigmatic case of empirical research on the language of thought and inferential role semantics // *Journal of Foreign Languages*. 2013. Vol. 36. No. 6. P. 27–41.
- O'Brien, 2014 – *O'Brien D.P.* Conditionals and disjunctions in mental-logic theory: A response to Liu and Chou (2012) and to López-Astorga (2013) // *Universon*. 2014. Vol. 29. No. 2. P. 221–235.
- Peirce, 1931–1958 – *Peirce C.S.* *Collected papers of Charles Sanders Peirce* / Ed. by C. Hartshorne, P. Weiss, A. Burks. Cambridge, Mass.: Harvard University Press, 1931–1958, 8 vols.
- Quelhas et al., 2017 – *Quelhas A.C., Rasga C., Johnson-Laird P.N.* A priori true and false conditionals // *Cognitive Science*. 2017. Vol. 41. No. 55. P. 1003–1030.



Rips, 1994 – *Rips L.J.* The Psychology of Proof: Deductive Reasoning in Human Thinking. Cambridge, Mass.: Massachusetts Institute of Technology (MIT) Press, 1994. 463 p.

Von Wright, 1951 – *Von Wright G.H.* Deontic logic // *Mind*. 1951. Vol. 60. No. 237. P. 1–15.

Von Wright, 1956 – *Von Wright G.H.* A note on deontic logic and derived obligation // *Mind*. 1956. Vol. 65. No. 260. P. 507–509.

Von Wright, 1963 – *Von Wright G.H.* Norm and Action. A logical Enquiry. L.: Routledge & Kegan Paul, 1963. 214 p.

References

Braine, M. D. S., O'Brien, D. P. (eds.). *Mental Logic*. Mahwah, NJ: Lawrence Erlbaum Associates, Inc., Publishers, 1998. 481 pp.

Cheng, P. W., Holyoak, K. J. Pragmatic reasoning schemas, *Cognitive Psychology*, 1985, no. 17, pp. 391–416.

Cheng, P. W., Holyoak, K. J. On the natural selection of reasoning theories, *Cognition*, 1989, no. 33, pp. 285–313.

Fodor, J. Why are so good at catching cheaters, *Cognition*, 2000, no. 75, pp. 29–32.

Forrester, J. W. *Being Good and Being Logical: Philosophical Groundwork for a New Deontic Logic*. New York: M. E. Sharpe, Inc, 1996. 332 pp.

Gouveia, E. L., Roazzi, A., Moutinho, K., Bompastor Borges Dias, M. G., O'Brien, D. P. Raciocínio condicional: influências pragmáticas, *Estudos de Psicologia*, 2002, vol. 7, no. 2, pp. 217–225.

Henlé, M. On the relation between logic and thinking, *Psychological Review*, 1962, no. 69, pp. 366–378.

Johnson-Laird, P. N. Against logical form, *Psychologica Belgica*, 2010, vol. 5, no. 3/4, pp. 193–221.

Johnson-Laird, P. N. Inference with mental models, in: K. J. Holyoak, R. G. Morrison (eds.) *The Oxford Handbook of Thinking and Reasoning*. New York: Oxford University Press, 2012, pp. 134–145.

Johnson-Laird, P. N., Girotto, V., Legrenzi, P. Modelli mentali: Una guida facile per il profane, *Sistemi Intelligenti*, 1999, vol. XI, no. 1, pp. 63–84.

Johnson-Laird, P. N., Khemlani, S., Goodwin, G. P. Logic, probability, and human reasoning, *Trends in Cognitive Sciences*, 2015, vol. 19, no. 4, pp. 201–214.

Johnson-Laird, P. N., Khemlani, S., Goodwin, G. P. Response to Baratgin et al.: Mental models integrate probability and deduction, *Trends in Cognitive Science*, 2015, vol. 19, no. 10, pp. 548–549.

López-Astorga, M. The disjunction introduction rule: Syntactic and semantic considerations, *Pragmalingüística*, 2015, no. 23, pp. 141–149.

López-Astorga, M. The first rule of Stoic logic and its relationship with the *indemonstrables*, *Tópicos. Revista de Filosofía*, 2016, no. 50, pp. 9–23.

O'Brien, D. P. Conditionals and disjunctions in mental-logic theory: A response to Liu and Chou (2012) and to López-Astorga (2013), *Universum*, 2014, vol. 29, no. 2, pp. 221–235.



O'Brien, D. P., Li, S. Mental logic theory: A paradigmatic case of empirical research on the language of thought and inferential role semantics, *Journal of Foreign Languages*, 2013, vol. 36, no. 6, pp. 27–41.

C. Hartshorne, P. Weiss, A. Burks (eds.). *Peirce C. S. Collected papers of Charles Sanders Peirce*. Cambridge, Mass.: Harvard University Press, 1931–1958, 8 vols.

Quelhas, A. C., Rasga, C., Johnson-Laird, P. N. A priori true and false conditionals, *Cognitive Science*, 2017, vol. 41, no. 55, pp. 1003–1030.

Rips, L. J. *The Psychology of Proof: Deductive Reasoning in Human Thinking*. Cambridge, Mass.: Massachusetts Institute of Technology (MIT) Press, 1994. 463 pp.

Von Wright, G. H. Deontic logic, *Mind*, 1951, vol. 60, no. 237, pp. 1–15.

Von Wright, G. H. A note on deontic logic and derived obligation, *Mind*, 1956, vol. 65, no. 260, pp. 507–509.

Von Wright, G. H. *Norm and Action. A logical Enquiry*. London: Routledge & Kegan Paul, 1963. 214 pp.